PROBLEM SET 2

It's OK to co-operate with classmates on problem sets. If you get stuck on a problem, don't waste a lot of time on it --- you have better things to do.

The following problems from Starr's *General Equilibrium Theory*, *2nd edition*, are assigned.

11. 2

12.7

12.8

23.4

In addition, two problems adapted from past quals are assigned, attached below.

This question is taken from the September 2010 Micro Qual.

4. There is a long, but useful introduction to the question. The actual question appears at the bullet, •, below. Subscripts indicate the vector co-ordinate; superscripts indicate the name of a firm or an exponent.

$Y^j \subset \mathbf{R}^N$ is firm j's technology set, negative co-ordinates representing inputs, positive co-ordinates representing outputs. The population of firms is represented as $j \in F$.

$Y \equiv \sum_{j \in F} Y^j$ is the aggregate technology set of the economy as a whole.

$r \in \mathbf{R}^N_+$ represents the initial resource endowment of the economy. $y^* \in Y^j$ is said to be *attainable* if there is $y^k \in Y^k$, $k \in F$, $k \neq j$ so that $y^* + \sum_{k \in F, k \neq j} y^k \geq -r$. The inequality holds co-ordinatewise.

Recall the following assumptions on production in the Arrow-Debreu general equilibrium model.

(P.I) Y^j is convex for each $j \in F$.

(P.II) $0 \in Y^j$ for each $j \in F$.

(P.III) Y^j is closed for each $j \in F$.

(P.IV) (a) if $y \in Y$ and $y \neq 0$, then $y_k < 0$ for some k.

(b) if $y \in Y$ and $y \neq 0$, then $-y \notin Y$.

(P.V) For each $j \in F$, Y^j is strictly convex.

Assume P.I, P.II, P.III, and P.IV. Choose a positive real number c, sufficiently large so that for all $j \in F$, $|y^j| < c$ (a strict inequality) for all y^j attainable in Y^j.

Let $\tilde{Y}^j = Y^j \cap \{y \in \mathbf{R}^N \mid |y| \leq c\}$. Note the weak inequality in the definition of $\tilde{Y}^j$ and the strong inequality in the definition of c.

Define the restricted supply function of firm j as

$$\tilde{S}^j(p) = \{y^{*j} \mid y^{*j} \in \tilde{Y}^j, p \cdot y^{*j} \geq p \cdot y^j \text{ for all } y^j \in \tilde{Y}^j\}$$

Define the (unrestricted) supply function of firm j as

$$S^j(p) = \{y^{*j} \mid y^{*j} \in Y^j, p \cdot y^{*j} \geq p \cdot y \text{ for all } y \in Y^j\}$$

Note that $S^j(p)$ may not exist (may not be well defined).

Theorem Assume P.II, P.III, P.IV, and P.V. Let $p \in \mathbf{R}^N_+$, $p \neq 0$. Then

(a) $\tilde{S}^j(p)$ is a well-defined (nonempty) continuous (point-valued) function, and

(b) if $\tilde{S}^j(p)$ is attainable in Y^j, then $\tilde{S}^j(p) = S^j(p)$.

- Let $N = 2$. Consider the production function for firm j, $G(x) = x^2$ (that is, x squared). Restated in Arrow-Debreu notation $Y^j = \{(-x, x^2) \mid x \in \mathbf{R}_+\}$. The Theorem above does not correctly apply to this Y^j.

  Explain how firm j fails to fulfill the assumptions of the theorem. Explain what results of the theorem are no longer reliable.

This question is taken from the September 2011 Micro Qual.

3. Consider a small economy, with two goods and three households. The two goods are denoted x, y. The households have identical preferences on $\mathbf{R}_+^2$ described by the utility function

$$u(x,y) = \sup[x,y]$$

where "sup" indicates the supremum or maximum of the two arguments. These tastes could be characterized by the household saying

> I like x and y equally well, and more is definitely better. But they are redundant. When there's more x, I use the x and discard the y. And when there's more y, it's y that I use and discard the x."

The households have identical endowments of (10,10).

(i) Demonstrate that there is no competitive equilibrium in this economy [Hint: Show that price vector $(\frac{1}{2}+\varepsilon, \frac{1}{2}-\varepsilon)$, $\varepsilon > 0$ cannot be an equilibrium; similarly for $(\frac{1}{2}-\varepsilon, \frac{1}{2}+\varepsilon)$; and finally $(\frac{1}{2}, \frac{1}{2})$. That pretty well takes care of it.]

(ii) The standard results for an Arrow-Debreu general equilibrium model include proofs of existence of general equilibrium. That result apparently fails in the example above. Explain. How can this happen? Is the example above a counterexample, demonstrating that the usual existence of general equilibrium results are invalid? Does the example above fulfill the usual sufficient conditions for existence of general equilibrium in an Arrow-Debreu model?